1) $x_1 = 2.47 \quad x_2 = 2.46 \quad x_3 = 2.61 \quad x_4 = 2.37 \quad x_5 = 2.45 \quad x_6 = 2.42 \quad x_7 = 2.37 \quad x_8 = 2.36$

$x_9 = 2.53 \quad x_{10} = 2.54 \qquad n = 10 \qquad X \sim N(0, \sigma^2) \qquad \sigma = 0.12$

$\sigma^2 = 0.0144$

(i)

$\alpha = 0.05 \qquad H_0)\ m = 2.5 \qquad C = \left\{ |\bar{X} - m_0| > \frac{\sigma}{\sqrt{n}} q_{1-\frac{\alpha}{2}} \right\}$

$\bar{X} = \frac{1}{n} \sum_i x_i = \frac{24.58}{10} = 2.458 \qquad n = 10 \qquad q_{1-\frac{\alpha}{2}} = q_{0.975} = 1.96$

$m_0 = 2.5$ accettata se $|\bar{X} - m_0| \leq \frac{\sigma}{\sqrt{n}} q_{1-\alpha/2}$ quindi:

$|2.458 - 2.5| \leq \frac{0.12}{\sqrt{10}} \cdot 1.96 \longrightarrow 0.042 \leq \frac{0.2352}{3.16228} \longrightarrow 0.042 \leq 0.74$

Vero $\Longrightarrow H_0$ accettata

(ii)

Calcolare $\bar{\alpha}$ t.c. $\frac{\sqrt{n}}{\sigma} |X - m_0| = q_{1-\alpha/2} \rightarrow \bar{\alpha} = 2\left[1 - \Phi\left(\frac{\sqrt{n}}{\sigma} |\bar{X} - m_0|\right)\right]$

$\bar{\alpha} = 2\left[1 - \Phi\left(\frac{\sqrt{10}}{0.12} \cdot |24.58 - 2.5|\right)\right] = 2 - 2\Phi(1.10679) \sim 2 - 1.72866 \sim$

~ 0.27

(iii)

$P_{m_0}(C) = 1 - P_{m_0}(A) = 1 - \Phi\left(\sqrt{n}\ \frac{(m_0 - m)}{\sigma} + q_{1-\alpha/2}\right) -$

$- \Phi\left(\sqrt{n}\ \frac{(m_0 - m)}{\sigma} + q_{1-\alpha/2}\right) = 0.45407$

2) $n = 500 \qquad \bar{x} = 39.5 \qquad S^2 = 25 \qquad H_0)\ m = 40$

$S = \sqrt{S^2} = 5$

(i)

$\alpha = 0.03 \qquad 1 - \frac{\alpha}{2} = 0.985 \qquad C = \left\{ |\bar{x} - m_0| > \frac{S}{\sqrt{n}} \tau_{(1-\frac{\alpha}{2}, n-1)} \right\}$

H_0 é accettata se $|\bar{X} - m_0| \leq \frac{S}{\sqrt{n}} \tau_{(1-\alpha/2,\, n-1)}$

$139.5 - 401 \leq \frac{5}{\sqrt{500}} \tau_{(0.985,\, 49)} \rightarrow 0.5 \leq \frac{5 \cdot 2.14315}{22.36068} \sim 0.47$ Falso

H_0 non é accettata

(ii)

$\bar{\alpha} = 2[1 - F_{n-1}(\frac{\sqrt{n}}{S}|\bar{X} - m_0|)] = 2 - 2F_{499}(\frac{\sqrt{500}}{5} \cdot 0.5) = 2 - 2F_{499}(2.236068)$
$= 2 - 2 \cdot 0.975 = 0.025 \longrightarrow$ poco plausibile (< 0.3)

3) $n = 1000$ $\quad \bar{X} = 50.5$ $\quad S^2 = 4$ $\quad \alpha = 0.02 \rightarrow 1 - \frac{\alpha}{2} = 0.99$

$S = 2$

$H_0)$ $m = m_0$

$C = \{ |\bar{X} - m_0| > \frac{S}{\sqrt{n}} \tau_{(1-\alpha/2,\, n-1)} \}$

H_0 accettate se $|\bar{X} - m_0| \leq \frac{S}{\sqrt{n}} \tau_{(1-\alpha/2,\, n-1)}$ ovvero se:

$-\frac{S}{\sqrt{n}} \tau_{(1-\alpha/2,\, n-1)} \leq \bar{X} - m_0 \leq \frac{S}{\sqrt{n}} \tau_{(1-\alpha/2,\, n-1)}$ ovvero se:

$m_0 \in [\bar{X} - \frac{S}{\sqrt{n}} \tau_{(1-\alpha/2,\, n-1)}, \bar{X} + \frac{S}{\sqrt{n}} \tau_{(1-\alpha/2,\, n-1)}] = 50.5 \pm \frac{2}{\sqrt{1000}} \cdot 2.3263 =$

$= 50.5 \pm 0.147128 \rightarrow [50.352872, 50.647128]$

4) $\sigma = 0.01$ $\quad x_1 = 3.142$ $\quad x_2 = 3.163$ $\quad x_3 = 3.155$ $\quad x_4 = 3.150$ $\quad x_5 = 3.141$

$n = 5$ $\quad \bar{X} = \frac{1}{n} \sum_i x_i = 3.1502$

(i)

$\alpha = 0.10$ $\quad H_0)$ $m \geq 3.152$

H_0 accettata se $\bar{X} - m_0 \geq \frac{\sigma}{\sqrt{n}} q_\alpha$ quindi:

$$3.1502 - 3.152 \geq \frac{0.01}{\sqrt{5}} \cdot q_{0.10} \longrightarrow 0.0018 \geq \underbrace{-q_{90}}_{\text{negativo}} \cdot \frac{0.01}{\sqrt{5}}$$

H_0 accettata

(ii)

$$\bar{\alpha} = \Phi\left(\frac{\sqrt{n}}{\sigma}(\bar{x} - m_0)\right) = \Phi(-0.4029) = 1 - 0.65542 = 0.344$$

(iii)

$$P_m(C) = 1 - P_m(A) = 1 - P_m\left(\frac{\sqrt{n}}{\sigma}(\bar{x} - m_0) \geq q_\alpha\right) = \Phi\left(\sqrt{n}\,\frac{m_0 - m}{\sigma} - q_\alpha\right) =$$
$$= \Phi(-0.8378) = 1 - \Phi(0.84) = 0.200$$

5) $x_1 = 3.048$ $x_2 = 2.970$ $x_3 = 3.120$ $x_4 = 3.140$ $x_5 = 3.010$ $n = 5$

$$\bar{x} = \frac{1}{n}\sum_i x_i = 3.0576$$

(i)

$\alpha = 0.10 \rightarrow 1 - \alpha/_2 = 0.95$ $H_0)$ $m \geq 3.02$

$$C = \left\{ |\bar{x} - m_0| > \frac{S}{\sqrt{n}} \tau_{(1-\alpha/_2, n-1)} \right\}$$

H_0 è accettata se $|\bar{x} - m_0| \leq \frac{S}{\sqrt{n}} \tau_{(1-\alpha/_2, n-1)}$ quindi:

$$S^2 = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2 = \frac{1}{4} \sum_{i=1}^{5} x_i^2 - 5\bar{x}^2 = 0.005 \rightarrow S = \sqrt{0.005} = 0.07$$

$$|3.0576 - 3.02| \leq \frac{0.07}{\sqrt{5}} \cdot 2.1318 \longrightarrow 0.0376 \leq 0.0667$$

H_0 accettata

(ii)

$$\bar{\alpha} = 1 - F_{n-1}\left(\frac{\sqrt{n}}{S} (\bar{x} - m_0) \right) = 1 - F_4(1.201) = 1 - 0.87 = 0.13$$

Poco plausibile ($\bar{\alpha} < 0.3$)

6) $x_1 = 6.66 \quad x_2 = 6.72 \quad x_3 = 6.68 \quad x_4 = 6.80 \quad x_5 = 6.66 \quad x_6 = 6.68 \quad x_7 = 6.66$

$x_8 = 6.82 \quad x_9 = 6.76 \quad x_{10} = 6.68 \qquad n = 10 \qquad \bar{x} = \frac{1}{n} \sum_i x_i = 6.712$

(i)

$\alpha = 0.05 \qquad H_0)\ \sigma \leq 0.04 \longrightarrow \sigma^2 \leq 0.04^2$

$$S^2 = \frac{1}{n-1} \sum_i (x_i - \bar{x})^2 = \frac{1}{9} \sum_i x_i^2 - 10\bar{x}^2 = 0.0037$$

$$C = \left\{ \sum_i^n \frac{(x_i - \bar{x})^2}{\sigma^2} = (n-1)\frac{S^2}{\sigma^2} > \chi^2_{(1-\alpha,\, n-1)} \right\}$$

H_0 accettata se $(n-1)\frac{S^2}{\sigma^2} \leq \chi^2_{(1-\alpha,\, n-1)}$ quindi:

$9 \cdot \frac{0.0037}{0.0016} \leq 16.9190 \longrightarrow 20.8125 \leq 16.9190$ H_0 non accettata

(ii)

$$\bar{\alpha} = 1 - G_{n-1}\left(\sum_i \frac{(x_i - \bar{x})^2}{\sigma^2} \right) = 1 - G_9(20.8125) = 1 - 0.98 = 0.02$$

Poco plausibile ($\bar{\alpha} < 0.3$)

7) $n = 10000 \qquad X_k \sim B(0, p)$

(i)

$\alpha = 0.05 \longrightarrow 1 - \alpha/2 = 0.975 \qquad H_0)\ p = \frac{1}{2}$

$C = \left\{ \frac{\sqrt{n}\,|\bar{x} - p|}{\sqrt{p(1-p)}} > q_{1-\alpha/2} \right\}$ H_0 accettata se $\frac{\sqrt{n}\,|\bar{x} - p|}{\sqrt{p(1-p)}} \leq q_{1-\alpha/2}$ quindi:

$\bar{x} = \hat{p} = \frac{5106}{10000} = 0.5106$ e $q_{1-\alpha/2} = q_{0.975} = 1.96$ $\quad p(1-p) = \frac{1}{4}$

$\frac{100\,|0.5106 - 0.5|}{\sqrt{0.25}} \leq 1.96 \longrightarrow 2.12 \leq 1.96$ $\quad$ *H_0 non accettata*

(ii)

$\bar{\alpha} = 2\left[1 - \Phi\left(\frac{\sqrt{n}}{\sqrt{p(1-p)}}\,|\hat{p}-p|\right)\right] = 2 - 2\Phi\left(\frac{100}{\sqrt{0.25}}\,|0.5106 - 0.5|\right) =$

$= 2 - 2\Phi(2.12) = 2 - 2 \cdot 0.983 = 0.034$

(iii)

$\bar{\alpha} = 2\left[1 - \Phi\left(\frac{\sqrt{n}}{\sqrt{p(1-p)}}\,|\hat{p}-p|\right)\right] \geq 0.3$

$1 - \frac{0.3}{2} \geq \Phi\left(\frac{\sqrt{n}}{\sqrt{p(1-p)}}\,|\hat{p}-p|\right) \longrightarrow q_{0.85} \geq \frac{\sqrt{n}}{\sqrt{p(1-p)}}\,|\hat{p}-p| \longrightarrow$

$\hat{p} \leq \frac{q_{0.85}\sqrt{p(1-p)}}{\sqrt{n}} + p \longrightarrow \hat{p} \leq 1.035 \cdot \frac{\sqrt{0.25}}{100} + 0.5 = 0.505175$

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8) $X_k \sim B(1, p)$ $\quad n = 100$

(i)

$H_0)\ p \leq \frac{1}{6}$ $\quad \hat{p} = \bar{x} = \frac{17}{100} = 0.17$

$\bar{\alpha} = 1 - \Phi\left(\frac{\sqrt{n}}{\sqrt{p(1-p)}}(\bar{x} - p)\right) = 1 - \Phi\left(\frac{\sqrt{100}}{\sqrt{\frac{5}{36}}}(0.17 - 0.16)\right) \sim 1 - \Phi(0.0894)$

$\sim 1 - 0.53387 = 0.46613$ $\quad$ *Molto plausibile* $(\bar{\alpha} > 0.3)$

(ii)

$\alpha = 0.4$ $\quad$ L'ipotesi é accettate in quanto $\bar{\alpha} > \alpha$