1) $\{X_1, \dots, X_{14}\}$ $\quad \bar{X} = 13 \quad \bar{\sigma}^2 = 2 \quad X_k \sim N(m, \sigma^2)$

(i)

$$I = \bar{X} \pm \frac{S}{\sqrt{n}} \tau_{(1-\frac{\alpha}{2}, n-1)}$$

$1-\alpha = 0.98 \Rightarrow \alpha = 0.02$

$q_{1-\frac{\alpha}{2}} = q_{0.99} \sim 2.6503$

$$I = 13 \pm \left(\frac{1}{7}\right)^{1/2} \tau_{(0.99, 13)} = 13 \pm \left(\frac{1}{7}\right)^{1/2} \cdot 2.6503 = 13 \pm \frac{2.6503}{\sqrt{7}} = 13 \pm 1.0017$$

$$= [11.998, 14.0017]$$

(ii)

$q_\alpha = -q_{(1-\alpha)}$

$$I = (-\infty, \bar{X} + \frac{S}{\sqrt{n}} \tau_{(1-\alpha, n-1)}]$$

$$I = 13 + \left(\frac{1}{7}\right)^{\frac{1}{2}} \tau_{(0.98, 13)} = 13 + \frac{2.3237}{\sqrt{7}} \sim 13.8783$$

(iii)

$$I = [\bar{X} - \frac{S}{\sqrt{n}} \tau_{(\alpha, n-1)}, +\infty)$$

$$I = 13 - \left(\frac{1}{7}\right)^{1/2} \tau_{(0.02, 13)} = 13 + \left(\frac{1}{7}\right)^{1/2} \tau_{(0.98, 13)} = 13 + \frac{2.3237}{\sqrt{7}} \sim 13.8783$$

2) $1-\alpha = 0.98 \rightarrow \alpha = 0.02 \rightarrow \frac{\alpha}{2} = 0.99$

(i)

Sto cercando n t.c. $\frac{\sqrt{p(1-p)}}{\sqrt{n}} q_{1-\frac{\alpha}{2}} = \frac{\sqrt{p(1-p)}}{\sqrt{n}} q_{0.99} \sim$

$$\frac{\sqrt{p(1-p)}}{\sqrt{n}} \cdot 2.33 < 0.05 \iff n > 20^2 \cdot p(1-p)(2.33)^2$$

$p(1-p)$ al massimo è $\frac{1}{4}$

$$n > 100 \cdot (2.33)^2 \sim 542$$

(ii)

$\bar{x} < 0.3$ $\qquad \hat{p} = \bar{x}$ quindi:

$$\frac{\sqrt{p(1-p)}}{\sqrt{n}} q_{1-\frac{\alpha}{2}} \sim \frac{\sqrt{\bar{x}(1-\bar{x})}}{\sqrt{n}} \cdot 2.33 < 0.05 \rightarrow n > 400 \cdot \bar{x}(1-\bar{x})(2.33)^2$$

massimizzando: $n > 400 \cdot 0.3(1-0.3)(2.33)^2 = 84\,(2.33)^2 \sim 456$

3) $x_1 = 6.68$ $\quad x_2 = 6.76$ $\quad x_3 = 6.78$ $\quad x_4 = 6.74$ $\quad x_5 = 6.64$ $\quad x_6 = 6.81$

$n = 6$ $\qquad 1-\alpha = 0.99 \rightarrow \alpha = 0.01$

$$I_{dx} = \left[(n-1)\frac{S^2}{\chi^2_{(1-\alpha, n-1)}}, +\infty\right) \qquad I_{sx} = \left(0, (n-1)\frac{S^2}{\chi^2_{(\alpha, n-1)}}\right]$$

(i)

$$\bar{x} = \sum_{x_i} \frac{x_i}{n} = 6.735 \qquad S^2 = \sum_{i=1}^{n} \frac{(X_i - \bar{X})^2}{n-1} = 0.0041 \rightarrow S = 0.064$$

$$\chi^2_{(0.99, 5)} = 15.0863 \longrightarrow I_{dx} = \left[\frac{5 \cdot 0.0041}{15.0683}, +\infty\right) = [0.00135, +\infty)$$

$$\chi^2_{(0.01, 5)} = 0.5534 \longrightarrow I_{sx} = \left(0, \frac{5 \cdot 0.0041}{0.5534}\right] = (0, 0.36713]$$

4) $x_1 = 3.142$ $\quad x_2 = 3.163$ $\quad x_3 = 3.155$ $\quad x_4 = 3.150$ $\quad x_5 = 3.141$

$1-\alpha = 0.95 \longrightarrow \alpha = 0.05 \rightarrow \frac{\alpha}{2} = 0.025$ $\qquad X_k \sim N(m, \sigma^2)$ con $m = 0$

$\sigma = 0.01$

$\sigma^2 = 0.0001$ $\qquad \bar{x} = \frac{1}{n} \sum_{x_i} x_i = 3.1502$

(i)

$$I = \bar{x} \pm \frac{\sigma}{\sqrt{n}} q_{1-\frac{\alpha}{2}} = 3.1502 \pm \frac{0.01}{\sqrt{5}} \cdot 1.96 = [3.14143, 3.15897]$$

(ii)

Trovare $1-\alpha$ t.c. $\dfrac{q_{(1-\alpha/2)} \frac{\sigma}{\sqrt{n}}}{\bar{x}} = 10^{-3}$

$$q_{(1-\alpha/2)} = \frac{10^{-3} \cdot \bar{x}\sqrt{n}}{\sigma} = \frac{10^{-3} \cdot 3.1502\sqrt{5}}{0.01} = 0.7044 \rightarrow 1-\frac{\alpha}{2} = 0.75804$$

$\rightarrow 1-\alpha \sim 0.52$

(iii)

Trovare n t.c. $\dfrac{q_{(1-\alpha/2)} \frac{\sigma}{\sqrt{n}}}{\bar{x}} = 10^{-3}$

$$n = \left(\frac{q_{(1-\alpha/2)}\, \sigma}{\bar{x} \cdot 10^{-3}} \right)^2 \longrightarrow n \geq 39$$

5) $X_k \sim B(1,p)$ $\quad \bar{x} = \hat{p} = P(X_k > 100) = \dfrac{350}{1000} = 0.35$

$1-\alpha = 0.99 \rightarrow \alpha = 0.01 \longrightarrow \alpha/2 = 0.02$

$$I = \hat{p} \pm \frac{\sqrt{\hat{p}(1-\hat{p})}}{\sqrt{n}}\, q_{(1-\alpha/2)} = 0.35 \pm \frac{\sqrt{0.35 \cdot 0.65}}{\sqrt{1000}} \cdot 2.575 = 0.35 \pm 0.0388$$

$I = [0.31116, 0.38884]$