Dimostrazione della relazione tra entropia congiunta e condizionata

Dimostriamo $H(X,Y) = H(X) + H(Y \mid X)$.

$$\begin{aligned}
H(X,Y) &= -\sum_{i=1}^{h}\sum_{j=1}^{k} p(x_i, y_j) \log p(x_i, y_j) \\
&= -\sum_{i=1}^{h}\sum_{j=1}^{k} p(x_i, y_j) \log(p(x_i)p(y_j \mid x_i))) \\
&= -\sum_{i=1}^{h}\sum_{j=1}^{k} p(x_i, y_j)(\log p(x_i) \log p(y_j \mid x_i)) \\
&= -\sum_{i=1}^{h}\sum_{j=1}^{k} p(x_i, y_j) \log p(x_i) - \sum_{i=1}^{h}\sum_{j=1}^{k} p(x_i, y_j) \log p(y_j \mid x_i) \\
&= H(X) - \sum_{i=1}^{h}\sum_{j=1}^{k} p(x_i)p(y_j \mid x_i) \log p(y_j \mid x_i) \\
&= H(X) - \sum_{i=1}^{h} p(x_i) \sum_{j=1}^{k} p(y_j \mid x_i) \log p(y_j \mid x_i) \\
&= H(X) + H(Y \mid X)
\end{aligned}$$

Analogamente si ottiene $H(X,Y) = H(Y) + H(X \mid Y)$.