

$$|A_1 \times A_2 \times \cdots \times A_r|$$

$$|A_1 \times A_2 \times \cdots \times A_r| = \prod_{i=1}^r |A_i|$$

quindi

$$|A^n| = |A|^n$$

Dimostrazione

Caso base

$$|A_1 \times A_2| = |A_1| \cdot |A_2|$$

Passo induttivo

$$\begin{aligned} |A_1 \times A_2 \times \cdots \times A_{r-1} \times A_r| &= |(A_1 \times A_2 \times \cdots \times A_{r-1}) \times A_r| \\ &= \prod_{i=1}^{r-1} |A_i| \cdot |A_r| = \prod_{i=1}^r |A_i| \end{aligned}$$