Generatori di $U + W$

Se $U = \text{Span}(u_1, \dots, u_h), W = \text{Span}(w_1, \dots, w_k) \subseteq V$ spazio vettoriale, allora $U + W = \text{Span}(u_1, \dots, u_h, w_1, \dots, w_k)$.

Dimostrazione

$U + W \subseteq \text{Span}(\dots)$:

$$u + w = (\alpha_1 u_1 + \dots + \alpha_h u_h) + (\beta_1 w_1 + \dots + \beta_k w_k) \in \text{Span}(\dots)$$

$\text{Span}(\dots) \subseteq U + W$:

$$v \in \text{Span}(\dots) \implies v = \underbrace{\alpha_1 u_1 + \dots + \alpha_h u_h}_{\in U} + \underbrace{\beta_1 w_1 + \dots + \beta_k w_k}_{\in W} \in U + W$$